Table 2 Effect of expansion of driver gas through an area ratio of $\frac{1}{4}$

a_4/a_1	P_4/P_1	M_s	a_4'/a_1	P_4'/P_1	M_s'
3	155,000	9.3	2.5	62,000	7.6
4	155,000	11.7	3.3	62,000	9.7
5	155,000	13.7	4.2	62,000	11.5
6	155,000	15.6	5.0	62,000	12.9
8	155,000	19.0	6.7	62,000	15.6
10	155,000	22.1	8.4	62,000	17.9

sistance gage that was placed in the end flange. The resulting signals were displayed on an oscilloscope that permitted the determination of the maximum testing time $t_E - t_C$.

The experimental and ideal results are presented in Fig. 2. The latter were obtained as follows. Knowing the Mach number of the incident shock wave we may obtain the time of arrival of the shock wave at the end wall t_C and the time t_D when the shock wave, which is reflected from the end wall, encounters the contact surface. Then, if we know the velocity of the shock wave that is reflected from the contact surface (back to the end wall), we may obtain the quantity $t_E - t_D$. Adding this result to the quantity $t_D - t_C$ gives the ideal value for the maximum testing time.

To obtain the velocity of the second reflected shock wave, we must know the pressure ratio across the wave P_6/P_5. This may be determined by iteration.[7]

The inner diameter of the driver section was 4 in. To increase the driver gas temperature, the region adjacent to the diaphragm (on the upstream side) has a steel insert that reduces the cross section to a 1-in. square. When the diaphragm is broken, the driver gas expands from the 1-in. square section through a conically shaped transition piece to the 2-in.-diam expansion section. The expansion reduces the temperature and pressure of the driver gas, which results in a reduction in the Mach number of the shock wave. This effect was studied by Lin and Fyfe,[8] and we have used their results to obtain an estimate of the reduction in shock strength. The results are presented in Table 2 for an initial pressure ratio across the diaphragm P_4/P_1 of 155,000. The primed quantities represent the effective values for the driver gas for a driver-area to expansion-area ratio of $\frac{1}{4}$. The results indicate that the expansion of the driver gas through an area ratio of $\frac{1}{4}$ produces a significant reduction in Mach number.

A shock Mach number of 9.5 in helium (21,000 fps) has been attained in the Harvard free piston shock tube, a sufficiently strong shock to produce ionization in helium at a pressure of 1-atm behind the reflected shock (approximately 15,000° K according to equilibrium conditions).

References

[1] Stalker, R. J., "An investigation of free piston compression of shock tube driver gas," National Research Council of Canada, MT-44, Ottawa (1961).

[2] Greif, R., "The free piston shock tube," Thesis, Harvard Univ. (1962).

[3] Duff, R. E., "Shock tube performance at low initial pressure," Phys. Fluids **2**, 207–216 (1959).

[4] Roshko, A., "On flow duration in low-pressure shock tubes," Phys. Fluids **3**, 835–842 (1960).

[5] Hooker, W. J., "Testing time and contact zone phenomenon in shock tube flows," Phys. Fluids **4**, 1451–1463 (1961).

[6] Strehlow, R. A. and Cohen, A., "The limitations of the reflected shock technique for studying fast chemical reactions and its application to the observations of relaxation in nitrogen and oxygen," Ballistic Research Labs. Rept. 1059, AD 209962, Maryland (1958).

[7] Ford, C. A. and Glass, I. I., "An experimental study of one-dimensional shock-wave refraction," J. Aeronaut. Sci. **23**, 189–191 (1956).

[8] Lin, S. C. and Fyfe, W. I., "Low density shock tube for chemical kinetics studies," Phys. Fluids **4**, 238–249 (1961).

Mass Injection Contours for a Hypersonic Leading Edge at an Angle of Attack

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Mass injection through a rounded orifice at the stagnation point of a symmetric body at hypersonic speeds has been investigated experimentally by both Baron and Alzner[1] and Tucker,[2] and theoretically by Wang[3] and Eminton.[4]

The smoothly contoured orifice has the advantage of supplying a larger buffer distance at the points where heating is most serious. Moreover it can be treated theoretically such that the flow field is prescribed.

Most re-entry vehicles enter the atmosphere at some predetermined angle of attack. The present note calculates the asymmetric orifice contour for mass injection cooling for a parabolic leading edge at an angle of attack with respect to the freestream (Fig. 1).

A 'Newtonian' pressure distribution is imparted on the parabolic contact surface, which is inclined at an angle θ with the axis.

Using the coordinate system shown in Fig. 2, the Newtonian pressure is found to be

$$p = p_0 - \rho_\infty U_\infty^2 \sin^2[\tan^{-1}(R/y) - \theta - (\pi/2)] \qquad (1)$$

where p_0 is the impact pressure at point A. This expression is valid along EAB where the surface is in the windward side of the freestream.

We transform to parabolic coordinates defined by

$$(x + iy) = (R/2) + (1/2R)(\xi + i\eta)^2 \qquad (2)$$

From the equations of continuity

$$(\partial/\partial\xi)[(\xi^2 + \eta^2)^{(1/2)}\rho q_{(\xi)}] + (\partial/\partial\eta)[(\xi^2 + \eta^2)^{(1/2)}\rho q_{(\eta)}] = 0 \qquad (3)$$

and momentum

$$(\partial/\partial\xi)[(\xi^2 + \eta^2)^{(1/2)}\zeta q_{(\xi)})] + (\partial/\partial\eta)[(\xi^2 + \eta^2)^{(1/2)}\zeta q_{(\eta)}] = 0 \qquad (4)$$

one can obtain an equation for the stream function

$$\frac{1}{\rho}\frac{\partial}{\partial\xi}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial\xi}\right) + \frac{1}{\rho}\frac{\partial}{\partial\eta}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial\eta}\right) = -f(\psi)(\xi^2 + \eta^2) \qquad (5)$$

where $\zeta = \rho f(\psi)$ is the vorticity.

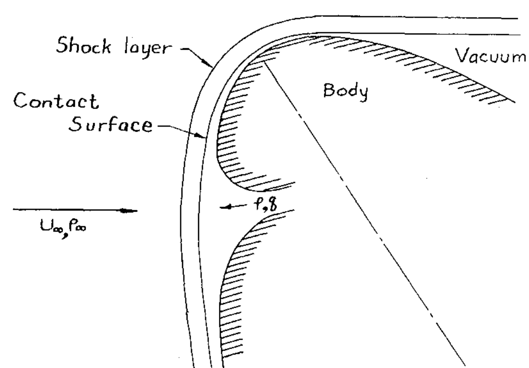

Fig. 1 Schematic diagram.

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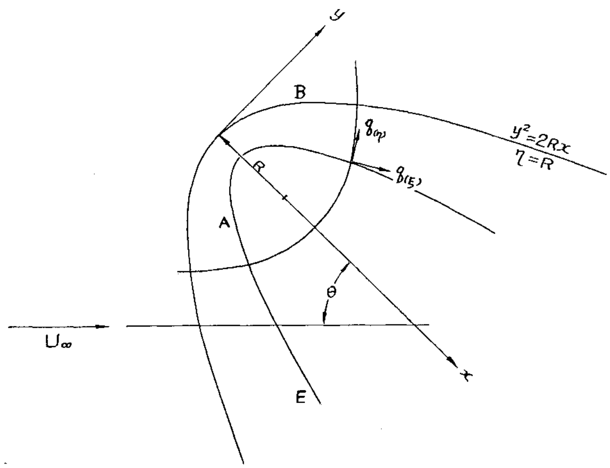

Fig. 2 Coordinate system.

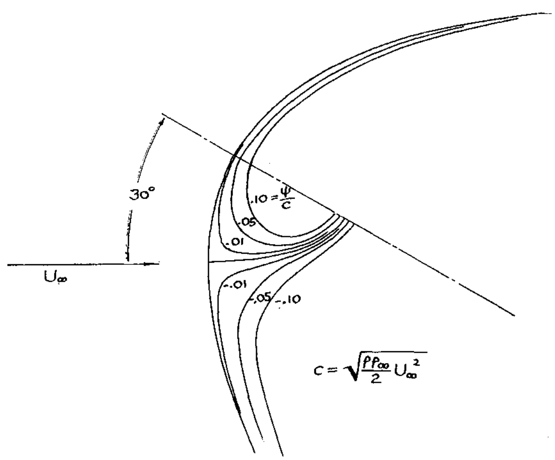

Fig. 3 Hypersonic injection contours for parabolic contact surface at 30° angle of attack.

For an irrotational flow field and a constant density approximation, Eq. (5) reduces to Laplace's equation as expected. With a suitable choice of constants, a stream function can be found which results in a pressure that matches Eq. (1) exactly, i.e.,

$$\psi = 2\left(\frac{\rho\rho_\infty}{2}U_\infty{}^2\right)^{(1/2)}\cos\theta\left(1-\frac{\eta}{R}\right)\left(\tan\theta+\frac{\xi}{R}\right) \quad (6)$$

The streamlines (or orifice contours) for $\theta = 30°$ and 45° are shown in Figs. 3 and 4. The rate of mass injected is $2R\psi$.

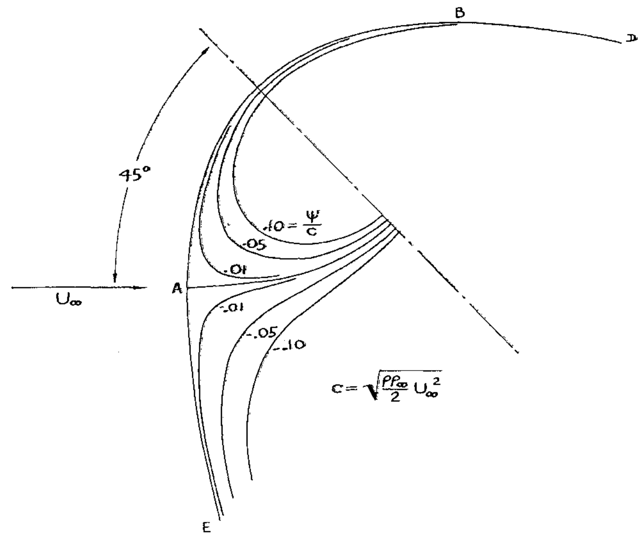

Fig. 4 Hypersonic injection contours for parabolic contact surface at 45° angle of attack (same scale).

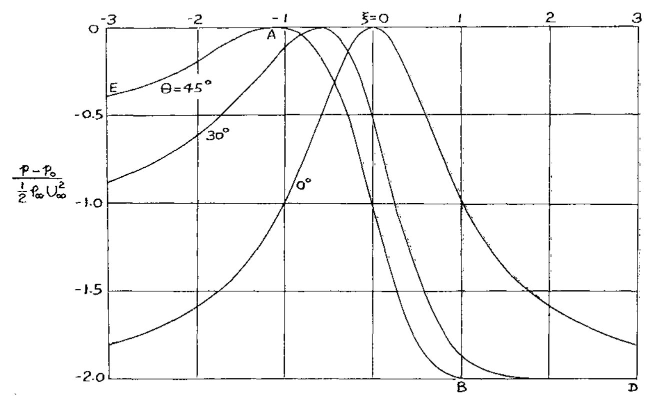

Fig. 5 Induced Newtonian pressure on the parabolic contact surface for angles of attack 0°, 30°, and 45°.

The corresponding Newtonian pressure distributions are shown in Fig. 5.

References

[1] Baron, J. R. and Alzner, E., "An experimental investigation of a two-layer inviscid shock cap due to blunt-body nose injection," J. Fluid Mech. **15,** 442–448 (1963).

[2] Tucker, L.M., "An Experimental investigation of contoured nose orifices with injection at free stream Mach numbers of 1.86 and 4.3," Royal Aircraft Establishment, TN Aero 2923 (1964).

[3] Wang, C. Y., "Contours for stagnation point mass injection in hypersonic flow," AIAA J. **2,** 178–179 (1964).

[4] Eminton, E., "Orifice shapes for ejecting gas at the nose of a body in two dimensional flows," Royal Aircraft Establishment, TN Aero 2711 (1960).